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PROFESSOR M. L. G. REDHEAD

FROM HEAD OF DEPARTMENT

30th September 1990

Dear Abner,

I am writing this letter on Mid-Atlantic, on my way to the Grünbaum Colloquia in Pittsburgh! Rob Clifton has shown me your reply to his letter dated 11th September, in which you stress again the corrections of Eq (16), in GHSZ. Since I am responsible for the omission of this equation let me explain in straightforward terms what I had in mind. First of all, Eq (16) follows with full rigour from Eq (11a) of GHSZ. This is essentially what you say in your letter, and I entirely agree. My disagreement is with the formulation of Eq (11a). As written in GHSZ it is assumed to be true for all λ. But all we

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can strictly infer is that the set of λ for which it is true has measure one. From this it follows that

for a given ϕ

$A_\lambda(2\phi) = A_\lambda(0)$ is true a.e. in λ.

We denote this equation by (16*)

Now my point is, one cannot derive (16) from (16*).

Denoting the measure on the Λ-space by m.

(16*) says $\forall\phi\ m(\{\lambda : A_\lambda(2\phi) = A_\lambda(0)\}) = 1$

while (16) says $\forall\phi\ \forall\lambda\ (A_\lambda(2\phi) = A_\lambda(0))$

From (16*) it certainly follows that

$$\forall\phi\ \exists\lambda\ (A_\lambda(2\phi) = A_\lambda(0))$$

But one cannot deduce from this

$$\exists\lambda\ \forall\phi\ (A_\lambda(2\phi) = A_\lambda(0)) \quad (*)$$

let alone (16).

For an uncountable infinity of ϕ's it is perfectly possible to have

$$\{\lambda : A_\lambda(2\phi) = A_\lambda(0)\} = \emptyset\ !$$

uncountable set of ϕ

which would illustrate the falsity of (*) and a fortiori of (16).

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FROM HEAD OF DEPARTMENT

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II

By choosing a countable or indeed a finite set of ϕ, all these problems are avoided, because

$$m\Big[\bigcap_{\text{countable set of } \phi} \{\lambda : A_\lambda(2\phi) = A_\lambda(0)\}\Big] = 1$$

so there are plenty of values of λ for which $A_\lambda(2\phi) = \text{const}$ for the countable set of ϕ.

whereas for uncountable ϕ, there may well be no λ for which $A_\lambda(2\phi) = \text{const}$. throughout this uncountable set.

For these reasons I cannot agree with your comment: "The proof in GHS2 is presented at the level of rigour common in mathematical theorems on functions of a real variable."

I hope (Pope) these remarks clear up what seems

to me a problem & mis-communication (if there is such a word!) between Yourself and Clifton.

With best wishes

Yours sincerely

Michael